A Revisit To Phase Noise Model Of Leeson

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Abstract—**Leeson's is one of the most famous models for predicting the phase noise in feedback oscillator. But there are several limitations and drawbacks. Leeson equation involves some key parameters, and these parameters are often determined by the oscillator structure and the oscillator circuit itself. Then a directly application of the Leeson's model without care would lead to erroneous results. Leeson's also assumed that the amplifier gain is remained a constant versus the frequency close the carrier frequency, and the filter transfer function is considered symmetrical on both sides of the carrier frequency. For a state of the art crystal oscillator the flat noise floor should be about -180dBc/Hz. The Leeson equation does not obviously describe these oscillators well, and the flat noise floor is about 15dB lower than that would be expected from the Leeson model.**

In this paper, a detailed analysis is performed to enlighten the key parameters and try to make it valid for all oscillator circuits. It explicitly takes needed parameters into account for phase-noise calculation, especially for the detailed descriptions of the flicker corner frequency (f_c) and the load Q-factor of oscillator circuits. In this paper, a phase flat noise floor model for oscillator is also described. This model allows us to specify that the flat noise floor is about -180dBc/Hz. An example of the phase noise of the colpitts quartz oscillator is also simulated and discussed.

I. Introduction

With the development of modern communication technology, the requirement of high stability and low phase noise oscillator has strongly increased, and one of the most important characteristics is phase noise spectrum. Phase noise is a specification that characterizes spectral purity. For instance, an oscillator output should ideally be a pure sinusoid represented as a vertical line, stationed on a single frequency, in the frequency domain. However, in practically, there are noise sources in the oscillator that can cause the output frequency to deviate from its ideal position, thus generating "unwanted" of other frequencies near the carrier frequency. The formatter will need to create these components, incorporating the applicable criteria that follow. These frequencies result from the noise sources modulating the oscillator. They often appear above the noise floor and close to the carrier frequency. Phase noise is usually specified as the ratio of a noise power at an offset frequency away from the carrier to the carrier power, in a 1-Hz bandwidth. Because the frequency of noise sources modulates the signal to produce phase noise, phase noise is unaffected by the slew rate.

Studies have been carried out to predict phase noise in oscillators in the past ten years. Different theories have also been put forward[1]-[4] for the modeling of the oscillator noise behavior. Although many models[5]-[10] have been developed for different types of oscillators, each of these models makes restrictive assumptions applicable only to a limited class of oscillators.

The most famous one is probably the Leeson model [5]-[6][9]-[10]. A mathematical analysis of this "heuristic" model has been proposed. But there are several limitations and drawbacks in this model. Firstly, Leeson equation involves some key parameters(especially for f_c and Q-factor), and these parameters are often determined by the oscillator structure, then a directly application of the Leeson model without care could lead to erroneous results. Secondly, Leeson's also assumed that the amplifier gain is remained a constant versus the frequency close the carrier frequency, and the filter transfer function is considered symmetrical on both sides of the carrier frequency. Thirdly, for a state of the art crystal oscillator the flat noise floor could be about -180dBc/Hz. The Leeson equation does not obviously describe these oscillators well, and the flat noise floor is about 15dB lower than that would be expected from the Leeson model.

In this paper, a detailed analysis is performed to enlighten the key parameters and try to make it valid for all oscillator circuits. It explicitly considers all the parameters needed for phase-noise calculation, especially for the detailed descriptions of f_c and the load Q-factor of oscillator circuit.

II. Brief Review Of Leeson Model And Some Conclusions

Leeson took oscillators as linear time invariant feedback systems[5] shown in Fig.1. A mathematical analysis of this "heuristic" model has been proposed[6]. The oscillator can be seen as an amplifier which has feed-back through a filter. If the gain is sufficient to overcome the filter attenuation and the phase shift is correct, oscillations will occur. If the amplitude of the oscillation is limited somehow, the amplifier can be made to operate in linear class A mode and then the Leeson's model will describe the main characteristics of the sideband noise.

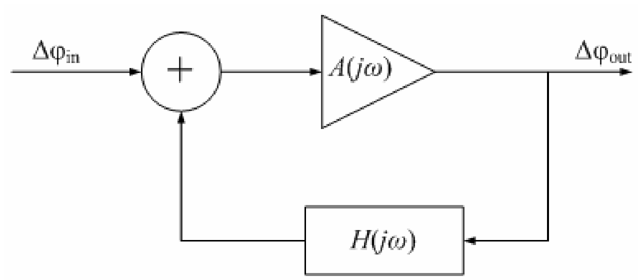

Figure 1. Leeson's Phase Noise Model of Feedback Oscillator

Leeson [5], using a single resonator feedback network, has derived the following expression that we called Leeson formula:

$$L(f_m) = 10\log\left[\left[\left(\frac{f_0}{2\times Q_L\times f_m}\right)^2+1\right]\times\frac{F\times k\times T}{P}\times\left(\frac{f_c}{f_m}+1\right)\right] \tag{1}$$

where f_m is offset frequency in Hz, $L(f_m)$ is noise level at f_m in dBc/Hz, f_0 is center frequency in Hz, Q_L is loaded Q, F is noise factor, T is absolute temperature, P is the carrier power in dBm, and f_c is corner frequency for flicker noise in Hz.

From the phase noise model of Leeson and G.Sauvage[6], we can draw some conclusions as follows:

(1) Phase noise (at a given offset frequency) improves with both the carrier power and Q_L increasing. These dependencies make sense. Increasing the signal power improves the ratio simply because the thermal noise is fixed, while increasing Q_L improves the ratio because the tank's impedance falls off as $1/(2\ Q_L f_m)$.

(2) At large frequency separations only the first line becomes non-zero. It says that the flat noise floor in dBc/Hz simply is the difference between the power delivered into the amplifier and the noise floor of the amplifier in dBm/Hz. Let's suppose a low noise figure like 5 dB and a very high power level into the amplifier like 3dBm one finds that the flat noise floor is expected no more than -174dBc/Hz.

(3) Close to the carrier, the bandwidth of the filter causes the noise that is produced at the amplifier output to be amplified with a positive feedback that depends on the frequency separation. The gain increases by 20dB/decade.

(4) At some frequency separation flicker noise will cause phase modulation. To some extent, the transistor amplifier is a phase modulator and the current variations through the transistor will change the phase shift through the amplifier very slightly. The flicker noise slopes at 10dB/decade and changes the slope from 20dB/decade to 30dB.

Using this model, it must be surprising that there are some significant differences between the spectrum predicted by (1) and what one typically measures in practice. There also have something that the model can not explain. The reasons should be like these:

- Leeson's model is based on linearization assumption, but oscillator is a nonlinear system, the linearization approximation would bring some big deviations.
- Leeson's model also assumed that the amplifier gain is remained a constant versus the frequency close the carrier frequency, and the filter transfer function is considered symmetrical on both sides of the carrier frequency.
- Leeson's ignores the conversion of AM noise into PM noise results from nonlinearities, or the amplitude frequency effect of a resonator, or the base-collector varactor effect [11].
- Leeson's involves some special parameters(such as f_c and Q_L), a directly application of the model without care would lead to erroneous results.
- A directly application of the Leeson's model is only expected no more than -174 dBc/Hz， it could not explain the flat floor noise about -180 dBc/Hz clearly.

III. CONSIDERATIONS ON PARAMETERS IN LEESON'S MODEL

For many years, in the field of linear feedback systems formalism, the Leeson formula is a very useful tool for the determination of phase noise in feedback oscillators. However, a successful application requires a careful identification of the parameters included in the formula according to the real oscillator structure and the circuit. This section will emphasize on how to application the parameters such as f_c and Q_L, and try to explain the reason why the flat noise floor of real state of the art oscillators can be made much better than predicted by the Leeson's model. There should be some considerations.

A. The Calculation of f_c

Flicker noise in BJTs is also known as $1/f$ noise because of the $1/f$ slope characteristics of the noise spectra. It is one of the components which contribute to oscillator phase noise, however, f_c should be determined by the whole oscillator circuit not the BJT itself, the expression is given like this[13]

$$f_c = \sqrt{\frac{Qf_0\alpha}{\beta}} = \sqrt{\frac{Qf_0\alpha p}{4kTF_N}} \tag{2}$$

Where α is flicker noise coefficient, β is white phase noise coefficient, P is the carrier power in dBm, T is absolute temperature, K is the Boltzmann's constant, and F_N is the respective noise factor.

B. The Calculation of Q_L

From the viewpoint of oscillator spectrum purity[14], the Q_L factor is defined as:

$$Q_L = \frac{\omega}{2}\left|\frac{1}{z(\omega)}\times\frac{dz(\omega)}{d\omega}\right|_{\omega=\omega_0} = \frac{\omega}{2}\left|\frac{d}{d\omega}\ln z(\omega)\right|_{\omega=\omega_0} \tag{3}$$

But for BJT oscillator, BJT is essentially a current controlled current source. Q_L should be defined like this:

$$Q_L = \frac{\omega}{2}\left|\frac{1}{z(\omega)}\times\frac{dz(\omega)}{d\omega}\right|_{\omega=\omega_0} = \frac{\omega}{2}\left|\frac{d}{d\omega}\ln\frac{z_{11}(\omega)}{z_{12}(\omega)}\right|_{\omega=\omega_0} \quad (4)$$

where $Z_{11}(\omega)$ and $Z_{12}(\omega)$ are terms of the circuit impedance matrix Z.

C. The Flat Floor of Phase Noise Spectrum in Oscillator

From Leeson formula, we can see that the flat floor noise should be no more than -174dBc. Because the output power of oscillators is often less than 0dBm. This power is relatively small, but state of the art oscillators can be made much better, as an example, here gives a type of 500-02268B oscillator (output Power>10dBm) made by www.wenzel.com. the detailed phase noise level is shown in Table I.

It seems that the Leeson's model can not explain the floor noise lower than -181dBc. However, if the power *P* in Leeson formula is defined as not the power sent into the amplifier but the power out of the amplifier, this contradiction could be explained.

As it is known, phase noise is a relative magnitude, not an absolute value. Absolute floor noise is determined by the product(in dBm) of the Boltzmann's constant and the absolute temperature. For the state of the art oscillators, it is not necessary to make the amplifier see the same source impedance at all frequencies, and its output power is much larger, then phase noise would be improved. In practice, when the amplifier is took into accounted and the current feedback is used, the noise at large frequency offsets will be much less amplified than the signal at the resonance frequency. Therefore, the ratio of signal to noise is increased, and the phase noise is improved as well.

Practically, there are amplitude variations and phase or frequency fluctuations in real oscillators. Previous investigations [12]-[13][15]-[16] often calculate the amplitude variations and the phase or frequency fluctuations separately. According to the definition, Phase noise is usually specified as the ratio of a noise power at an offset frequency away from the carrier to the carrier power, in a 1-Hz bandwidth. Numerical integration techniques could be used to incorporate the amplitude variations and phase or frequency fluctuations into mean-squared amplitude fluctuations $\Delta E^2(f)$ and the flat floor noise should be expressed like this[17]:

$$L(f)_{floor} = 10\lg\frac{\Delta E^2(f)}{V_0 B}(\text{dBc/Hz}) \quad (5)$$

where $\Delta E^2(f)$ is the mean-squared amplitude fluctuation measured at a Fourier frequency separation f, from the carrier in a measurement bandwidth B. (5) is often used in many measurement systems for flat floor noise measurement.

TABLE I. PHASE NOISE LEVEL OF 500-02268B

f_m	1Hz	10Hz	100Hz	1kHz	10kHz	100kHz
$L(f_m)$	−73	−103	−133	−161	−179	−181

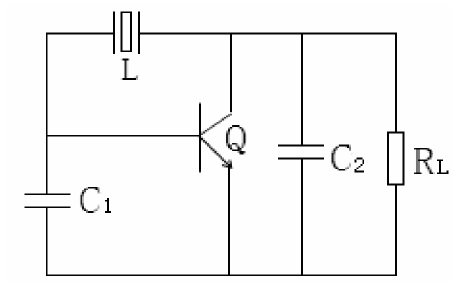

Figure 2. Colpitts Crystal Oscillator Prototype Circuit

IV. SIMPLE EXAMPLE: COLPITTS CRYSTAL OSCILLATOR

As an example, a modified 100MHzl crystal Colpitts oscillator is discussed. In this section, only Q_L of this circuit and its phase noise are considered. Fig.2 shows the equivalent circuit.

In the above circuit, the crystal resonator acts as an inductance. Here we suppose the impedance of resonator is $j\omega L$, the Z impedance matrix of passive network in this circuit should be expressed as follows[14]:

$$Z = \frac{1}{(1-\omega^2 LC_1)(1+j\omega C_2 R_L)+j\omega C_1 R_L} \times \begin{bmatrix} R_L + j\omega L - \omega^2 LC_2 R_L & R_L \\ R_L & R_L - \omega^2 LC_1 R_L \end{bmatrix} \quad (6)$$

And according to (4) and (6), the Q_L results in:

$$Q_L = \frac{\omega_0}{2}\left|\frac{R_L}{R_L + j\omega_0 L - \omega_0^2 LC_2 R_L}\times\left(\frac{jL}{R_L} - 2\omega_0 LC_2\right)\right| = \frac{\omega_0}{2}\sqrt{\frac{\omega_0^2 L^2\left(4\omega_0^2 C_2^2 R_L^2 + 1\right)}{R_L{}^2 + \omega_0^2 L^2 + \omega_0^4 L^2 C_2^2 R_L{}^2 - 2\omega_0^2 LC_2 R_L^2}} \quad (7)$$

From (7), we can draw a conclusion that Q_L would increase with C_2, that is to say, the larger C_2, the higher Q_L.

In this work, a 100MHz modified colpitts crystal oscillator is simulated, the equivalent circuit is shown in Fig.2 and the phase noise level is shown in Fig.3. The amplifier is took as one part of the oscillator in this simulation.

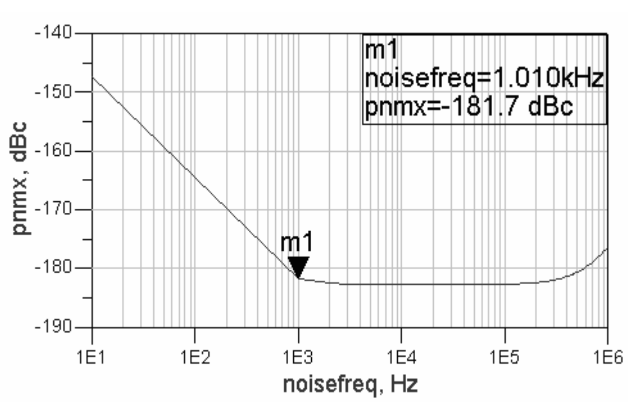

Figure 3. Simulated Phase Plot Of 100MHz Colpitts Oscillator

As we can see from Fig.3, the flat floor noise of this oscillator can reach up to -183dBc/Hz@1KHz with the ADS simulation software which designed by Agilent.

V. CONCLUSION

Phase noise is a very important characteristic of an oscillator. According to previous investigations, there are several key parameters that influence the phase noise. This paper based on a heuristic model called Lesson's model, and analyzed this model (including its useful conclusions and backwards). In the paper, discussion key parameters have been emphasized and general calculating expressions of these parameters in Leeson's model have been given. Then a prototype of colpitts crystal oscillator has also been simulated and discussed. There are still several questions should be considered in the further study.

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